WAVE OPTICS

Important Points:

1. The condition which allows us to use the principles of geometry is $b^2 >> l\lambda$

   Where b = size of the object interacting with light

   l = distance between the object and the screen

   λ = wavelength of light.

2. Two light sources are said to be coherent if they emit waves of same frequency which are in phase or which maintain a constant phase difference.

3. When two or more waves reach a point in space simultaneously, the resultant displacement at that point at any instant of time is the algebraic sum of the displacements produced by the individual waves. This is known as the principle of superposition.

4. If $\frac{b^2}{l} << \lambda$ Fraunhofer diffraction is observed.

5. If $\frac{b^2}{l} \cong \lambda$ Fresnel diffraction is observed

6. In interference, Resultant intensity

   $I = 4I_0 \cos^2 \frac{\phi}{2}$ (Where is maximum intensity of each individual wave)

   ϕ Is initial phase difference

7. Fringe width $\beta = \frac{\lambda D}{d}$

8. For two waves with intensities I_1 and I_2 with phase ϕ resultant intensity

   $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

9. When un-polarized light of intensity I_0 passes through a polarizer intensity of emergent light

   $I = \frac{I_0}{2}$

10. Bending of light at the edges of an obstacle or aperture is called diffraction. The phenomenon of encroachment of light into the geometrical shadow of an obstacle is known as diffraction.

11. Diffraction is exhibited by both transverse and longitudinal waves.

12. Diffraction confirms wave nature of light.

13. Diffraction is due to the superposition of waves originating from different points of the exposed portion of the same wave front.

14. Polarization of Light:

"The process of confining the vibrations of electric field vector of light into a single plane" is known as polarization of light

15. Plane of Vibration or Plane of Polarization:

The plane which contains the vibrations of electric field of light (polarized light) is known as plane of vibration or plane of polarization.

16. Law of Malus:

When the plane polarized light of intensity I_0 falls on a polarizer with an angle θ to the axis of polarizer, then intensity of refracted light $I = I_0 \cos^2 \theta$

17. Brewster'S Law:

For polarization by reflection $\mu = \tan i_p$

$\mu \rightarrow$ Refractive index of reflecting surface

$i_p \rightarrow$ Angle of polarization

Very Short Answer Questions

1. What is Fresnel Distance?

A. Fresnel Distance:

The distance beyond which divergence of the beam of width ‘a’ become significant is called Fresnel distance.

Fresnel distance $Z_F \approx \frac{a^2}{\lambda}$

a = size of the aperture

λ = wave length of light

2. Give the justification for validity of ray optics?

A. Fresnel distance $Z_F \leq \frac{a^2}{\lambda}$ is the validity of ray optics.

If the distance between aperture and screen much smaller than i.e., $\frac{a^2}{\lambda}$ diffraction pattern cannot be observed so ray optics is applicable.

3. **What is Polarisation of Light?**

A. The phenomenon in which vibration of light vector (electric vector) are confined to a particular direction is called polarisation.

4. What is Malus Law?

A. Malus Law:

The intensity of polarized light transmitted through the analyzer varies as the square of the cosine of the angle between the plane of transmission of the polarizer and analyzer.

$I = I_0 \cos^2 \theta$ Where θ is the angle between the axis of the polarizer and analyzer.

5. Explain Brewster's Law.

A. Brewster's Law:

The tangent of the angle of polarisation is equal to the refractive index of the reflecting medium.

$$\mu = Tan\, i_p$$

At polarising angle the reflected ray and refracted ray are perpendicular to each other.

6. When does a monochromatic beam of light incident on a reflective surface get completely transmitted?

A. If $Z_F = \frac{a^2}{\lambda}$ then diffraction pattern is not observed then monochromatic beam of light incident on a reflective surface gets completely transmitted.

Short Answer Questions

1. Explain Doppler Effect in light. Distinguish between Red Shift and Blue Shift?

A. Doppler Effect Light:

The apparent change in the frequency due to relative motion between the source and observer is called Doppler Effect.

If 'ν' is the actual frequency and 'ν'' is the apparent frequencies, then the relative change in frequency

$$\frac{\Delta\nu}{\nu} = -\frac{v}{c} \text{ or } \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Here 'c' is the speed of light and 'v' is the velocity of the source which is small compared to that of light. Doppler Effect in light is symmetric.

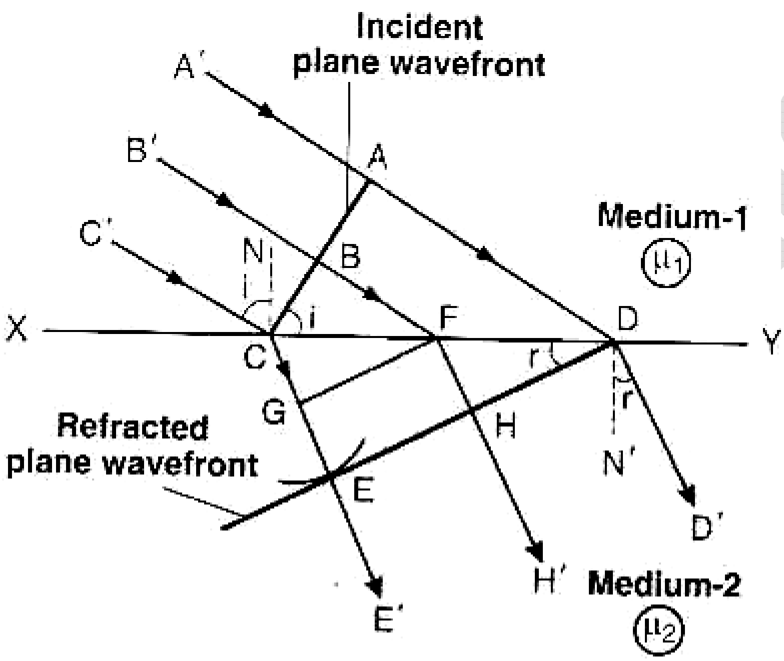
Red Shift and Blue Shift: - Apparent wavelength > Actual Wavelength.

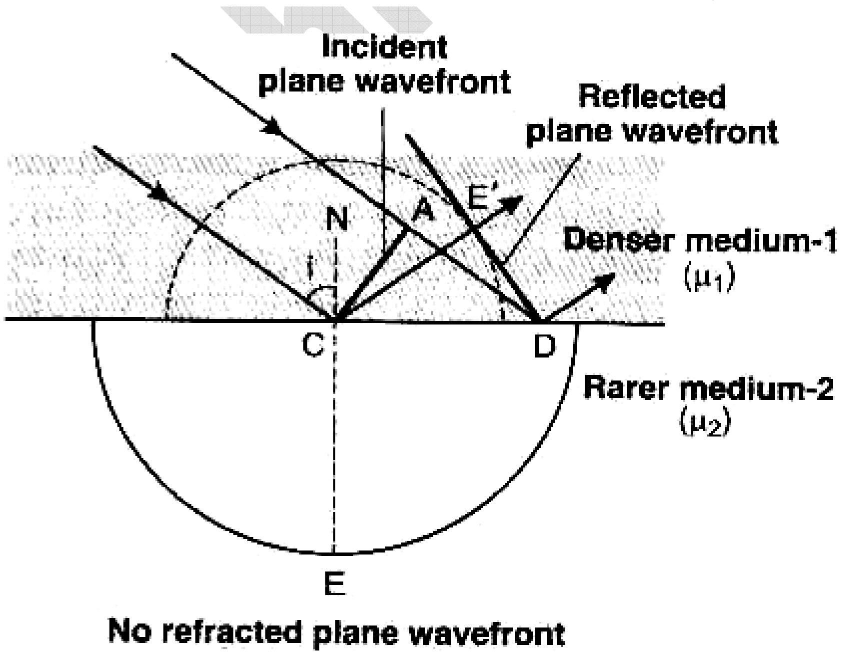
Hence the spectrum of the radiation from the source of light shifts towards the red end of spectrum. When a star is moving away from the Earth the wavelength increases and it looks more reddish. This is red shift phenomenon. When the waves are received from a source moving towards the observer, there is an apparent decrease in wavelength. This is called blue shift.

2. What is total internal reflection? Explain the phenomenon using Huygens principle?

A. Let XY be a surface separating the two media (1) and (2) of refractive indices μ_1 and μ_2 respectively and let V_1 and V_2 be the speed of light waves in medium (1) and medium (2) respectively. Let AC be a plane wave front incident on XY. Lines AA^1 and CC^1 which are normals to the incident plane wave front (i.e., AC) are called incident rays.

Huygens'principle:





If CN is the normal at the point C, then $\angle C'CN = i$ (angle of incidence). Angle of incidence is also the angle which the incident plane wave front makes with the plane XY, i.e. $\angle ACY = i$. The points A and C on this wave front will serve as the sources of disturbance and will give out secondary wavelets. During the time the secondary wavelet from A strikes the surface XY at D, the secondary wavelet from C would have travelled a distance CE in the medium-2 where the distance CE is such that time taken by the secondary wavelet to travel a distance AD in the medium-1 = time taken by the secondary wavelet to travel a distance CE in the medium-2. i.e.

$\frac{AD}{V_1} = \frac{CE}{V_2}$ (As time = distance/velocity)............ (i)

Total Internal Reflection

We have seen that the radius of the secondary spherical wavelet from C in medium-2 given by equation. (1)

$$\frac{AD}{CE} = \frac{V_1}{V_2} \text{ or } CE = \frac{V_2}{V_1} \times AD$$

$$CE = \left(\frac{\mu_1}{\mu_2}\right) CD \sin i \left(\text{as } \sin i = \frac{AD}{CD}\right)$$

CE > CD. In this case if a hemispherical wavelet is drawn with C as centre, the point D will lie inside this wavelet. Since no tangent plane can be drawn from D to this wavelet. Since no tangent plane can be drawn from D to this wavelet, there is no refracted plane wave front which implies that no refraction is possible. But a reflected wave front in the medium-1 is possible. This is due to the reason that the radius of the reflected hemispherical wavelet from C is equal to CE' (which is less than CD). This situation corresponds to total internal reflection.

3. Derive the expression for the intensity at a point where interference of light occurs. Arrive at the conditions for maximum and zero intensity?

Interference:

A. The redistribution of energy due to super imposition of two or more waves is called interference.

Derivation for Interference Pattern:

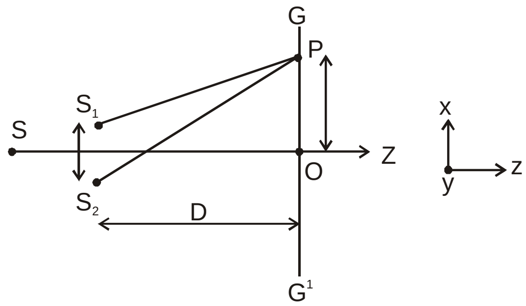
Consider two waves coming from S_1 and S_2

They interfere at a point 'P' on the screen. Their equations are given by

$Y_1 = a\sin\omega t$

$Y_2 = a\sin(\omega t + \phi)$

Where ϕ is the phase difference between two waves



The resultant wave equation is given by

$Y = Y_1 + Y_2$

$Y = a\sin\omega t + a\sin(\omega t + \phi)$

$= a\sin\omega t + a[\sin\omega t\cos\phi + \cos\omega t\sin\phi]$

$= a\sin\omega t(1+\cos\phi) + a\cos\omega t\sin\phi$

Let $a(1+\cos\phi) = R\cos\theta$ ------ (1)

$a\sin\phi = R\sin\theta$ ------ (2)

$Y = R\sin\omega t\cos\theta + R\sin\theta\cos\omega t$

$= R\sin(\omega t + \theta)$

The above equation represent's S.H.M and its amplitude is R

Squaring and Adding equations (1) and (2), we get

$R^2 = [a(1+\cos\phi)]^2 + [a\sin\phi]^2$

$= a^2[1+\cos^2\phi + 2\cos\phi + \sin^2\phi]$

$= a^2[2+2\cos\phi] = 2a^2[1+\cos\phi]^2$

$R^2 = 4a^2\cos^2\phi/2$

As intensity $I \alpha R^2$

So, $I = R^2 = 4a^2 \cos^2 \phi / 2$

Condition for maximum intensity or bright fringe

$\cos(\phi/2) = \pm 1$

$\phi = 0, 2\pi, 4\pi, 6\pi, 8\pi, ..., 2\pi$

Path difference $(\Delta x) = 0, \lambda, 2\lambda, ...n\lambda$, where $n = 0,1,2,3,....$

Maximum intensity at bright band $= I_{max} = 4a^2$

Condition for minimum intensity or dark fringe

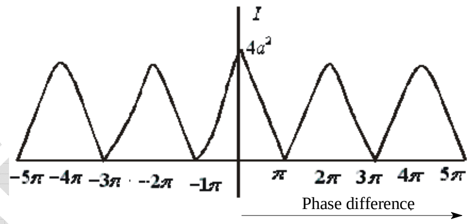
$I_{min} = 0 \Rightarrow 4a^2 \cos^2 \phi / 2 = 0$

$\Rightarrow \cos \phi / 2 = 0$

$\Rightarrow \phi = \pi, 3\pi, 5\pi, 7\pi, ...(2n-1)\pi$

Path difference $(\Delta x) = \dfrac{\phi\lambda}{2\pi} = \dfrac{\lambda}{0}, \dfrac{3\lambda}{2}, \dfrac{5\lambda}{2}, \dfrac{7\lambda}{2}, ... \dfrac{(2n-1)\lambda}{2}$, where $n = 1,2,3,.....$

The intensity varies between a maximum of I = $4a^2$ and minimum of zero but energy is conserved as shown in the figure.



4. Does the principle of conservation of energy hold for interference and diffraction phenomena? Explain briefly?

A. The principle of conservation of energy holds good for both interference and diffraction.

In case of interference the energy is distributed equally to all bright fringes as all the bright fringes are of equal intensity and all dark fringes are completely dark.

Where as a case of diffraction pattern, the bright fringes do not have equal intensity and dark fringes are not completely dark but less bright than bright fringes. Hence the energy is not distributed equally in case of diffraction but the energy is conserved.

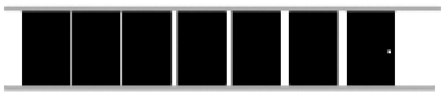
5. How do you determine the resolving power of your eye?

A. Resolving Power:

The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its resolving power.

Resolving Power of Eye:

Make black strips of equal width separated by white stripes. All the black strips should be of equal width, while that of white stripes should increase from left to right. For example let the black stripes have a width of 5 mm. Let the width of two white stripes be 0.5 mm each, the next two white stripes be 1 mm each, the next 1.5 mm each, etc. Paste this pattern on a wall in the room at the height of your eye.



Now watch the pattern with one eye. By moving away or closer to the wall, find the position where you can just see some black stripes as separate stripes. All the black stripes to the left of this stripe would merge into one another and would not be distinguished. The black stripes to the right of this would be more clearly visible. If 'd' is the width of the white stripe and 'D' is the distance of the wall from two crossed the eye. Then d/D is the resolution of the eye.

6. Discuss the intensity of transmitted light when a Polaroid sheet is rotated between two crossed Polaroids?

A. When un-polarized light is incident on a polarizer, the transmitted light is linearly polarized.

If this light further passes through analyser, the intensity varies with the angle between the transmission axes of polarized and analyser.

The intensity of the polarized light transmitted through the analyser is proportional to square of the cosine of the angle between the plane of transmission of analyser and the plane of transmission of polariser. This is known as Law of Malus.

$\therefore$ The intensity of polarized light after passing through analyser is $I = \frac{I_0}{2}\cos^2\theta$.

Where I_0 is the intensity of un-polarized light. $A = \frac{A_0}{\sqrt{2}}\cos\theta$ The amplitude of polarized light after passing through analyser.

Long Answer Questions

1. What is Huygens's principle? Explain the optical phenomenon of refraction using Huygen's Principle?

A. Huygens's Principle:-

a) Every point on the primary wave front gives out secondary wavelets.

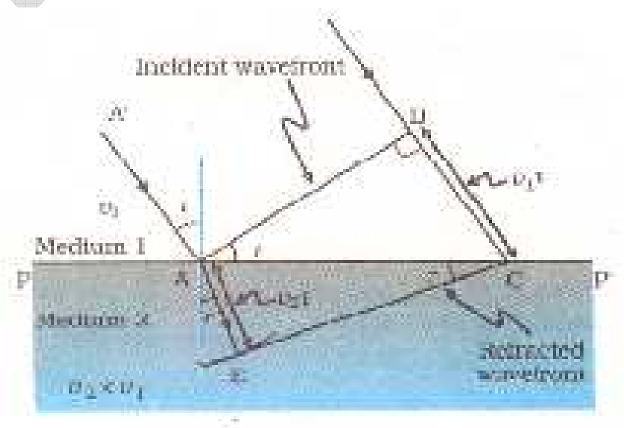
b) These secondary wavelets move in the forward direction with speed of light.

c) The position of the new wave front is obtained by drawing a tangent drawn to the edges of these wavelets at any instant.

Refraction:

Consider the plane wave front AB incident on a plane refracting surface PP^1 at A at an angle of incidence ' i ' as shown. The surface PP^1 separates a medium 1 and medium 2 in which velocities of light are V_1 and V_2 respectively.

Let the secondary wavelets from B reach PP^1 at C in a time t so that BC = $V_1 t$. During this time interval, the wavelets emitted from A travel a distance $V_2 t$. The new refracted wave front is EC.



In the right angle triangle ABC $\sin i = \frac{BC}{AC} = \frac{V_1 t}{AC}$

In the right angle triangle AEC, $\sin r = \frac{AE}{AC} = \frac{V_2 t}{AC}$

$\therefore \frac{\sin i}{\sin r} = \frac{V_1}{V_2} =$ Constant. Which proves the Snell's law of refraction.

If r < i , the speed of light in the second medium is less than that in the first medium..If speed of light in vacuum is C, then $n_1 = \frac{c}{v_1}$ and $n_2 = \frac{c}{v_2}$ which are the refractive indices of medium (1) and (2) respectively $n_1 \sin i = n_2 \sin r$. This is Snell's law of refraction.

If λ_1 and λ_2 are the wavelengths in of medium (1) and (2) respectively, then $\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{V_1}{V_2}$

Since the light travels faster in the rarer medium than in the denser medium, the wavelength of a light wave smaller in a denser medium than in a rarer medium.

2. Distinguish between coherent and incoherent addition of waves. Develop the theory of constructive and destructive interference?

A. **Coherent Sources:**

Two sources are said to be coherent if they emit the waves of same wave length and same amplitude and constant phase difference. The two slits illuminated by a single source, acts as coherent sources.

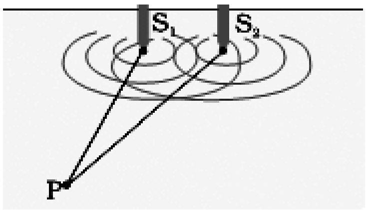
Incoherent Sources:

Two sources are said to be incoherent if they emit waves of different wave length, different amplitude and different phase

Two independent sources can never be coherent, even though wave length two amplitude are same they differ in phase.

Constructive and destructive interference:

Interference is based on the superposition principle according to which at a particular point in the medium, the resultant displacement produced by a number of waves is the vector sum of the displacements produced by each of the waves.



(a)

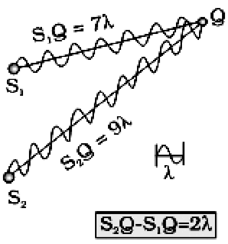
Consider two waves coming from s_1 and s_2.Consider a point P for which S_1 P = S_2 P If the displacement produced by the source S_1at the point P is given by $y_1 = a\cos\omega t$ then, the displacement produced by the source S_2 (at the point P) will be given by $y_2 = a\cos\omega t$

Thus, the resultant of displacement at P would be given by

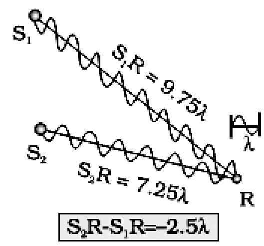
$$y = y_1 + y_2 = 2a\cos\omega t$$

Since the intensity is the proportional to the square of the amplitude, the resultant intensity will be given by $I = 4I_0$

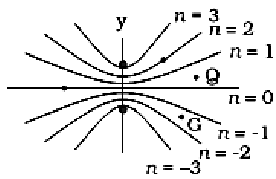
Where I_0 represents the intensity produced by each one of the individual sources; I_0 is proportional to a^2.



(a)



(b)



Locus of points for which $S_1P - S_2P$ is equal to zero, $\pm\lambda$, $\pm 2\lambda$, $\pm 3\lambda$.

(c)

Constructive Interference:

If two waves are moving in the same direction superimpose each other, at some points crest of one wave meets with crest of other. (Or) trough of one wave meets with trough of another i.e. two waves meet in phase then amplitude and intensity are maximum. This interference is called constructive interference.

In fact at any point on the perpendicular bisector of S_1S_2, the intensity will be $4I_0$.

The two sources are said to interfere constructively and we have what is referred to as constructive interference. From fig. (a).

$S_2Q - S_1Q = 2\lambda$

If we have two coherent sources S_1 and S_2 vibrating in phase, then for an arbitrary point P whenever the path difference,

$$S_1P \sim S_2P = n\lambda \ (n = 0, 1, 2, 3, ...)$$

We will have constructive interference and the resultant intensity will be $4I_0$; the sign ~ between S_1P and S_2P represents the difference between S_1P and S_2P.

Destructive Interference:

If two waves superimpose in opposite phase i.e. crest of one wave meet with trough of another then intensity and amplitude of the resultant wave is minimum. This interference is called destructive interference. From fig.(b)

$S_2R - S_1R = -2.5\lambda$

The two displacements are now out of phase and the two displacements will cancel out to give zero intensity. This is referred to as destructive interference.

On the other hand, if the point P is such that the path difference,

$$S_1P \sim S_2P = \left(n + \frac{1}{2}\right)\lambda \ (n = 0, 1, 2, 3, ...)$$

We will have destructive interference and the resultant intensity will be zero.

From Fig. (c) Let the phase difference between the two displacements be ϕ. Thus, if the displacement produced by S_1 is given by

$y_1 = a\cos\omega t$

Then, the displacement produced by S_2 would be

$y_2 = a\cos(\omega t + \phi)$

And the resultant displacement will be given by

$y = y_1 + y_2$

$= a\left[\cos\omega t + \cos(\omega t + \phi)\right]$

$= 2a\cos(\phi/2)\cos(\omega t + \phi/2)$

The amplitude of the resultant displacement is $2a\cos(\phi/2)$ and therefore the intensity at that point will be

$I = 4I_0 \cos^2(\phi/2)$

If $\phi = 0, \pm 2\pi, \pm 4\pi....$ which corresponds to the condition given by constructive interference leading to maximum intensity

On the other hand, if $\phi = \pm\pi, \pm 3\pi, \pm 5\pi....$ which corresponds to the condition given by destructive interference leading to zero intensity.

The positions of maxima and minima will not change with time. The positions of maxima and minima will also vary rapidly with time and a "time-averaged" intensity distribution. When this happens, we will observe an average intensity that will be given by

$< I >= 4I_0 < \cos^2(\phi/2) >$

Where angular brackets represent time averaging. $\cos^2(\phi/2)$ Will randomly vary between 0 and 1 and the average value will be 1/2. The resultant intensity will be given by $I = 2I_0$ at all points.

When the phase difference between the two vibrating sources changes rapidly with time, we say that the two sources are incoherent and when this happens the intensities just add up. This is indeed what happens when two separate light sources illuminate a wall.

3. Describe young's experiment for interference and hence arrive at the expression for 'fringe width'?

A. Consider two pin holes S_1 and S_2 on an opaque screen. The light waves spread out from the source 'S' and falls on both slits S_1 and S_2. Now the two slits S_1 and S_2. Behave like coherent sources.

From the source 'S' spherical wave fronts are produced. The spherical waves coming from S_1 and S_2 will produce interference fringes on the screen GG'. The point 'P' on the screen 'GG' is corresponding to a maximum intensity.

$S_2P - S_1P = n\lambda$, where $n = 0, 1, 2, 3, ...$

Now $(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(\frac{x+d}{2}\right)^2\right] - \left[D^2 + \left(\frac{x-d}{2}\right)^2\right]$

Where S_1S_2 = d and OP = x

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

If x, d<<<D then negligible error will be introduced.

So we can replace $S_2P + S_1P$ by 2D

$$S_2P + S_1P = 2D$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

$$S_2P - S_1P = \frac{xd}{D}$$

Hence we will have constructive interference resulting in a bright region.

When $x = x_n = \frac{n\lambda D}{d} . n = 0, \pm 1, \pm 2, ...$

We will have destructive interference resulting a dark region

When $x = x_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}; n = 0, \pm 1, \pm 2$

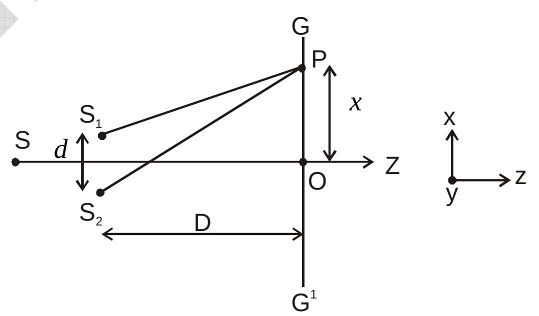
The dark and bright bands are called fringes.

The distances between two consecutive bright and dark fringes is called band width, which is given by

$$\beta = x_{n+1} - x_n$$

$$\beta = \frac{\lambda D}{d}$$

Which is the expression for the fringe width.



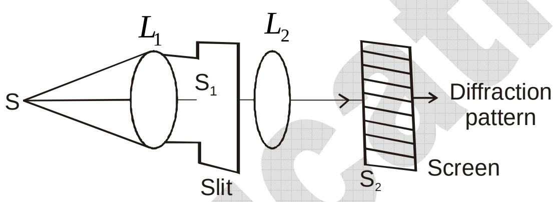
4. What is diffraction? Discuss diffraction pattern obtained from a single slit?

A. **Diffraction:**

The phenomenon of bending of light rays at the edges and corners of an obstacle and spreading of light into the geometrical shadow of the obstacle in called diffraction of light.

Fraunhoffer diffraction at a single slit:

Let S be a point source of light, emitting monochromatic light. The light rays emitted by the point source are made parallel with the help of a converging lens L_1. The emergent parallel rays are allowed to pass through a narrow rectangular slit S_1. The diffracted rays are focused on the screen S_2 using another convex lens L_2. The diffraction pattern is seen on the screen. The source, converging lenses, single slit and the screen are kept perpendicular to the plane of paper as shown in the figure.



Diffraction Pattern:

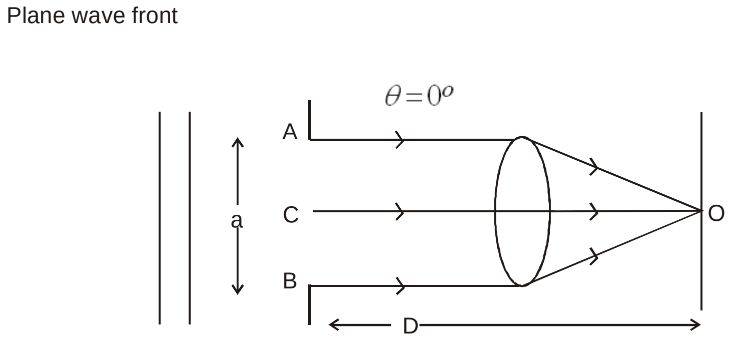
The non uniform distribution of light energy obtained on the screen due to the bending of light rays at the edges of the slit is called the diffraction pattern. The pattern consists of a broad and intense central maximum and a number of narrow and fainter maxima called secondary maxima on both sides of the central maxima. The width of the central maximum is twice as greater as that of the secondary maxima. In between the maxima then are minima called secondary minima.

Explanation: The maxima and the minima found in the diffraction pattern can be explained on the basis of Huygens principle. Each point on the incident wave front near the slit can be considered as a source of secondary wavelets. The resultant effect in a particular direction can be found by adding the secondary wavelets emitted in that direction using the principle of superposition. For simplicity let us consider the two dimensional case of the slit.

Theory of diffraction of light at single slit:

Let 'a' be the width of the slit 'λ'is the wavelength of monochromatic light used and 'D' is the distance of screen from the slit.

i) **Central Maximum:** All points on the wave front between A and B are in phase.



The second secondary minimum is located at another point P_2 when the path difference is 2λ i.e., $2\lambda = a\sin\theta_2$

Similarly of the path difference is 3λ i.e., $3\lambda = a\sin\theta_3$ and then third secondary minimum is obtained. In general for the n^{th} secondary minimum, we have difference = $n\lambda$

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

When n = 1, 2, 3.... But not n = 0 where there is a central bright fringe.

Positions of Secondary Maxima:

Between each pair of minima there is a secondary maximum (bright fringe).

The location of the first secondary maximum is given by $\frac{3\lambda}{2} = a\sin\theta_1^1$

Where θ_1^1 is the angle at which the rays are travelling with CO such that the ray from the top of the slit travels a distance $\frac{3\lambda}{2}$ more than the ray from the bottom edge.

The second secondary maximum is located on the screen when the path difference is $\frac{5\lambda}{2}$

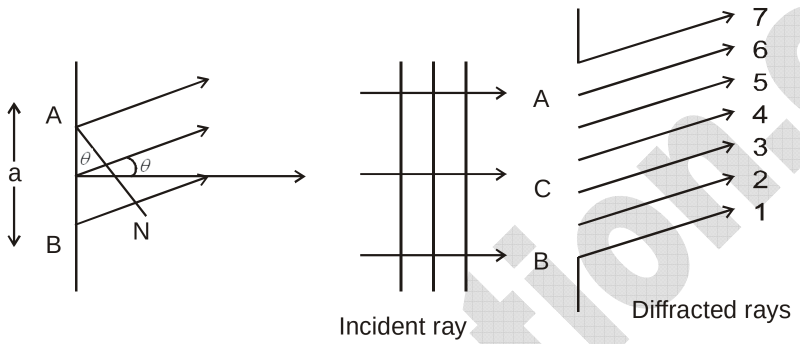
i.e., $\frac{5\lambda}{2} = a\sin\theta_2$

Consider a point 0 on the screen which lies on the perpendicular bisector of the slit as shown in fig.

The wavelets which fall on the lens parallel to CO $(ie\,\theta = 0^o)$ meet at point O in the same phase as there in no path difference between them. Thus all the waves arriving at O in phase give rise to central maximum is central bright fringe is obtained at 'O'.

Position of Secondary Minima:

Let us consider a point P_1 on the screen (fig) let the rays which reach the point P_1 make an angle θ_1 with CO. The rays from points A and B will have a path difference AN given by $AN = a\sin\theta_1$.



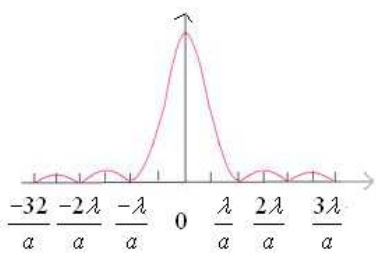
If the path difference is λ (the wave length of light used) then the point P_1 will have minimum intensity i.e., point P_1 is the first secondary minimum.

Then the location of the first secondary minimum is given by $\lambda = a\sin\theta_1$.

In general for the nth secondary maximum, the condition is

$a\sin\theta_n^1 = (2n+1)\frac{\lambda}{2}$ Where n = 1, 2, 3.....

The intensity of diffraction pattern of a single slit as a function of $\sin\theta$ is as shown in the figure



5. What is resolving power of Optical instrument? Derive the condition under which the images are resolved?

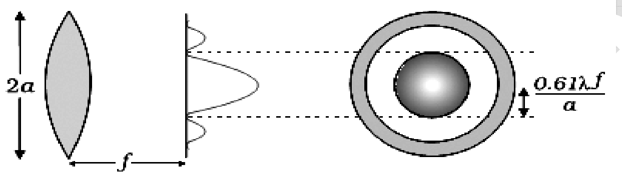
A. Resolving Power:

The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its Resolving Power.

Consider a parallel beam of light falling on a convex lens. If the lens is free from aberrations, the beam will get focused to a point. But due to diffraction the beam is focused to a spot of finite area. The pattern on the focal plane consists of a central bright region surrounded by concentric dark and bright rings .The radius of the central bright region is approximately

$$r_0 = \frac{1.22\lambda f}{2a} = \frac{0.61\lambda f}{a}$$

Where f is the focal length of the lens and 2a is the diameter of the lens.



Even though the size of the spot is very small, it plays an important role in determining the limit of resolution of optical instruments like a telescope or a microscope

Telescope:

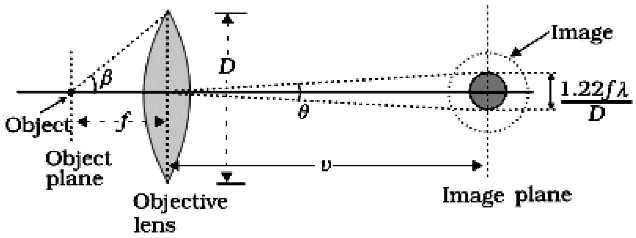
For two stars to be resolved

$$f\Delta\theta = r_0 = \frac{0.61\lambda f}{a} \Rightarrow \Delta\theta = \frac{0.61\lambda}{a}$$

The telescope will have better resolving power if ‘a’ is large. It is for this reason that for better resolution, a telescope must have a large diameter objective.

Microscope:

In the case of a microscope, the object is slightly beyond f and the magnification is .From the figure $D/f \approx 2\tan\beta$ where 2β is the angle subtended by the diameter of the objective lens at the focus of the microscope.



When the separation between two points in microscopic specimens is comparable to the wavelength of the light, the diffraction effects become important. The image of a point object

Will again be a diffraction pattern whose size in the image plane will be -

$$V\theta = V\left(\frac{1.22\lambda}{D}\right)$$

Two objects whose images are closer than this distance will not be resolved, they will be seen as one. The corresponding minimum separation, d_{min}, in the object plane is given by

$$d_{\min} = \left[V\left(\frac{1.22\lambda}{D}\right)\right]/m = \frac{1.22\lambda}{D}.\frac{V}{m} = \frac{1.22f\lambda}{D}$$

$$\therefore d_{\min} = \frac{1.22\lambda}{2\tan\beta} = \frac{1.22\lambda}{2\sin\beta} \qquad \left(\because D/f \approx 2\tan\beta\right)$$

If the medium between the object and the objective lens has refractive index ‘n’, then

$$d_{\min} = \frac{1.22\lambda}{2n\sin\beta}$$

The product $n\sin\beta$ is called the numerical aperture.

PROBLEMS

1. Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected and (b) refracted light? Refractive index of water is 1.33.

A. Wavelength of light $\lambda = 589\text{ nm} = 589\times10^{-9}\text{ m}$

Refractive index of water $\mu_w = 1.33$

(a) For Reflected Light

(i) Wavelength of reflected light $\lambda = 589\times10^{-9}\text{ m}$

(ii) Frequency of reflected light $\upsilon = \frac{c}{\lambda} = \frac{3\times10^{8}}{589\times10^{-9}}$

Where c is velocity of light ($\because$ Speed of light $c = 3\times10^{8}$ m/s)

$\upsilon = 5.09\times10^{14}$ Hz.

(iii) As the medium takes place in the same medium so,

Speed of reflected light $c = 3\times10^{8}$ m/s.

(b) For Refracted Light (In this process wavelength and speed changes but frequency remains the same)

Wavelength of refracted light $\lambda' = \frac{\lambda}{\mu} = \frac{589\times10^{-9}}{1.33} = 4.42\times10^{-7}\text{ m}$

Velocity of refracted light $v = \frac{c}{\mu} = \frac{3\times10^{8}}{1.33} = 2.25\times10^{8}\text{ m/s}$

2. What is the shape of the wave front in each of the following cases?

(a) Light diverging from a point source.

(b) Light emerging out of a convex lens when a point source is placed at its focus.

(c) The portion of the wave front of light from a distant star intercepted by the earth.

A. **(a)** Spherical wave front

(b) Plane wave front

(c) Plane wave front

3. (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0 x 10^8 m/s)

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

A. **(a)** Refractive index of glass $\mu_{glass} = 1.5$

Speed of light in vacuum c = 3 x 10^8 m/s

Speed of light in glass $v = \frac{c}{\mu} = \frac{3\times10^8}{1.5}$

$v = 2 \times 10^8$ m/s

(b) No, the speed of light is not independent of colour of light.

As we know that the refractive index of violet is greater than red.

$\mu_V > \mu_R$

So, velocity of violet is less than the velocity of red. Therefore, violet colour travels slower in glass, than the red colour

$v_V < v_R$

4. In a Young's double-slit experiment, the slits are separated by 0.28mm and the screen is placed 1.4m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2cm. Determine the wavelength of light used in the experiment?

A. $d = 0.28mm = 0.2810^{-3}m, D = 1.4m,$

$x_4 = 1.2cm = 1.2\times10^{-2}m$

P0sition of nth bright fringe,

$x_n = n\frac{D\lambda}{d}$ Or $x_4 = 4\frac{D\lambda}{d}$

$\therefore \lambda = \frac{x_4 d}{4D} = \frac{1.2\times10^{-2}\times0.28\times10^{-3}}{4\times1.4}$

$= 6\times10^{-7}m = 6000A^0.$

5. In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is k units. What is the intensity of light at a point where path difference is $\lambda/3$?

A. $I = 4I_o \cos^2 \frac{\phi}{2} = 4I_o = k$

Path difference $= \frac{\lambda}{3}$

Phase difference $= \frac{2\pi}{3}$

$$I = 4I_o \cos^2 \frac{\phi}{2} = 4I_o \cos^2 \pi/3 = I_o = \frac{k}{4}$$

6. A beam of light consisting of two wavelengths (650 nm and 520 nm is used to obtain interference fringes in a Young's double-slit experiment;

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm?

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelength coincide?

A. Wavelength $\lambda_1 = 650\,\text{nm} = 650 \times 10^{-9}$ m

And $\lambda_2 = 520\,\text{nm} = 520 \times 10^{-9}$ m

(a) For third fringe bright, n = 3

The distance of third bright fringe from central maximum

$$x = \frac{n\lambda D}{d} = 3 \times 650 \times 10^{-9} \times \frac{D}{d}\,\text{m} = \frac{3 \times 650 \times 10^{-9} \times 1.2}{2 \times 10^{-3}} = 1.17 \times 10^{-3}\,\text{m}\ .$$

(b) Let nth bright fringe due to wavelength λ_2 = 520 nm, coincide with $(n + 1)^{th}$ bright fringe due to wavelength λ_1 = 650 nm.

i.e., $n\lambda_2 \frac{D}{d} = (n-1)\lambda_1 \frac{D}{d}$

$$n \times 520 \times 10^{-9} = (n-1) 650 \times 10^{-9}$$

n = 5.

The least distance $x = n\lambda_2 \frac{D}{d} = 5\times 520\times 10^{-9}\frac{D}{d}$

$$x = 2600\frac{D}{d}\times 10^{-9}\,m$$

$$= 2600\times\frac{1.2\times 10^{-9}}{2\times 10^{-3}}\,m = 1.56\times 10^{-3}\,m = 1.56\text{ mm.}$$

7. In a double-slit experiment the angular width of a fringe is found to be 0.2^0 on a screen placed 1m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3?

A. Angular fringe width.

$$\theta = \frac{\beta}{D} = \frac{D\lambda/d}{D} = \frac{\lambda}{d} \quad \text{Or} \quad d = \frac{\lambda}{\theta} = \frac{\lambda^1}{\theta^1}$$

$$\text{or} \quad \theta' = \frac{\lambda'}{\lambda}.\theta = \frac{\lambda/\mu}{\lambda}.\theta = \frac{\theta}{\mu} = \frac{0.2^\circ}{4/3} = 0.6^\circ$$

8. What is Brewster angle (Refractive index of glass = 1.5) for air to glass transition?

A. From Brewster law, $\tan i_p = \mu = 1.5$

$\therefore$ Brewster angle, $i_p = \tan^{-1}(1.5) = 56.3^\circ$

9. Light of wavelength $5000\overset{0}{A}$ falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

A. Wavelength of reflected light = Wavelength of incident light

Or $\lambda = 5000^\circ = 5\times 10^{-7}\,m$

Frequency of reflected light,

$$v = \frac{c}{\lambda} = \frac{3.0\times 10^8}{5\times 10^{-7}} = 6\times 10^{14}\,Hz$$

By law of reflection, $\angle i = \angle r$

Given $\angle i + \angle r = 90°$

$\therefore \angle i = 45°$

10. Estimate the distance for which ray optics is a good approximation for an aperture of 4mm and wavelength 400nm?

A. $d = 4mm = 4\times10^{-3} m, \quad \lambda = 400nm = 4\times10^{-7} m$

$$D_F = \frac{d^2}{\lambda} = \frac{\left(4\times10^{-3}\right)^2}{4\times10^{-7}} = 40m.$$

Hence ray optics is valid up to a distance of 40m from the aperture.